Quantum complexity theory and proofs of advantage

This course aims to present the rigorous theoretical foundation of the notion of quantum advantage. The main quantum computational models will be introduced in detail as well as the associated quantum complexity classes. We will cover the known theoretical separation between quantum and classical from discrete to continuous variable quantum computing, providing a large panorama of the benefit we can expect from quantum technologies when it comes to theoretical computational complexity.

Prerequisites: linear algebra, basics of quantum statistical inference, elementary complexity theory.

Introduction to quantum complexity theory:
(Titouan Carette: 6 hours lecture + 6 hours TD)

- Computational models: Quantum Turing machines and quantum circuits, variations, gate set, approximate universality, Solovay-Kitaev.
- Complexity classes: BQP, QMA, QIP, known inclusions and equalities.
- Quantum query complexity: query model, polynomial bound, adversary bound, separation classical/quantum.

Analog model of quantum computation with continuous variable systems:
(Marco Fanizza: 3 hours lecture + 1 TD)

- States on separable Hilbert spaces, L^2(R^m), canonical commutation relations, harmonic oscillator, Fock space.
- Gaussian states and Gaussian operations, universal gate sets, classical simulability.
- Towards advantage with continuous variables: non-linearities, photon counting

Proofs of quantum advantage:
(Cambyse Rouzé: 6 hours lecture + 2 hours TD)

- More on decision problems, the polynomial hierarchy
- Counting problems and the class #P
- Sampling problems and the connection to counting
- Quantum advantage with Boson Sampling